HEAT EXCHANGE IN A MICROSTRUCTURAL FLUID UNDER BOUNDARY CONDITIONS OF THE THIRD KIND

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The problem of forced convection in a microstructural fluid in a plane channel under thermal boundary conditions of the third kind is solved. It is shown that taking account of the microstructure of the fluid leads to a decrease in the values of the temperature in the channel and the heat flow at the wall.

The fluid flowing in many heat-exchange systems is cooled or heated by the use of another fluid flowing in the channels of the system. As was noted in [1], in a number of cases the application of boundary conditions of the third kind is a good approximation for calculation of heat exchange in such systems when it is necessary to solve a coupled problem.

We consider the problem of heat exchange in a heat-conducting micropolar fluid (MPF) that is flowing in a plane channel, the theory of which was proposed in [2]. We neglect the axial heat conduction, energy dissipation, and compressibility of the fluid, and also the body forces and moments. We assume the physical properties of the MPF to be constant and the flow to be stabilized. Let the temperature of the flowing fluid and the temperature of the MPF in the initial section equal, respectively, T_1 and T_0. We are given the local coefficient of heat transfer from the inner surface of the wall to the surrounding medium K'. The y axis of the assumed coordinate system is perpendicular to the planes of the channel at a distance of 2h from each of them and the x axis coincides with the central line of the channel. The MPF flows under the action of a constant pressure gradient dP/dx, where the orientation of the microparticles coincides with the z axis.

For a given flow geometry the term connected with microrotations does not enter into the heat equation, as is the case in general [2]. In the problem being considered the microstructure of the fluid proves to have an effect on heat exchange by means of the variation of the hydrodynamic velocity profile, and for the description of heat exchange with the above assumptions it is necessary to solve the system of equations

$$(\mu+\varkappa)\frac{d^2v_x}{dy^2}+\varkappa\frac{d\nu_z}{dy}=\frac{dP}{dx}, \tag{1}$$

$$\gamma\frac{d^2\nu_z}{dy^2}-\varkappa\left(\frac{dv_x}{dy}+2\nu_z\right)=0, \tag{2}$$

$$v_x\frac{\partial T}{\partial x}=a\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

In studies concerning the hydrodynamics of a MPF, there has not yet been a solution of the question of the most acceptable physically valid boundary conditions for the vector of microrotation. If for the translational velocity **v** we always assume a boundary condition of adhesion consisting of the equality of **v** on the solid surface to the velocity at the same boundary, then for the microrotation we assumed various types of boundary conditions.

Mostly, for the solution of the problems we assume the so-called condition of total adhesion, for which on the solid–MPF boundary the velocity **v** and the microrotation $\boldsymbol{\nu}$ coincide with the velocity **V** and the angular velocity $\boldsymbol{\Omega}$ of the boundary [2-9]:

$$\mathbf{v}|_\Gamma=\mathbf{V},\quad \boldsymbol{\nu}|_\Gamma=\boldsymbol{\Omega}. \tag{4}$$

However, it is natural to assume that the particles of the MPF in the wall layer are all pulled toward the rigid boundary. This assumption received experimental confirmation in [10].

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Analytically solving the problem of the flow of blood in a cylindrical capillary, Ariman et al. [11] proposed for $\mathbf{v}$ the boundary condition

$$\left[\frac{1}{r}\frac{d}{dr}(r\mathrm{v}_\varphi)\right]_{|\Gamma} = 0. \tag{5}$$

The use of this condition leads to a velocity profile that is in good agreement with the experimental results of [10]. It is evident that conditions of types (4) and (5) are only particular cases: the first denotes the maximum effect of the wall on the rotation of particles, when the latter do not rotate, and the second denotes the absence of moment stresses at the boundary.

The maximum contribution to the development of physically applicable boundary conditions of general form was introduced in [12], in which a dynamic condition is proposed, according to which the magnitude of the microrotation is proportional to the surface density of the micromoments:

$$(\alpha_{ik}\mathrm{v}_k)_{|\Gamma} = (m_{ik}n_k)_{|\Gamma}. \tag{6}$$

Here the case $\alpha_{ik} \to \infty$ corresponds to total adhesion (4), and $\alpha_{ik} \to 0$ corresponds to a condition of type (5).

Recently, Kirwan and Newman obtained a generalization of a single boundary condition [13]. It consists of equating $\mathbf{v}$ on the boundary to an arbitrary constant quantity ν_w, on which is superimposed only the symmetry condition on the opposite boundaries. In the problem being considered this is

$$\mathrm{v}_z(h) = -\mathrm{v}_W, \quad \mathrm{v}_z(-h) = \mathrm{v}_W. \tag{7}$$

In our opinion, in the majority of problems on the flow of a MPF it is not necessary to resort to this form of the boundary conditions, since it leads to a complex analysis of the dependence on its solution.

In [14] a kinematic condition of type

$$\mathbf{v}_{|\Gamma} = \frac{\alpha}{2}(\mathrm{curl}\,\mathbf{V})_{|\Gamma} \tag{8}$$

was proposed, where $0 \leqslant \alpha \leqslant 1$. The conditions (6), (7), and (8), which are different in form, have the same physical sense – they indicate the presence on the boundary of microrotations, the magnitude of which can be different from the vortex and either is given or is determined by the corresponding coefficients. However, condition (8) admits an especially simple, physical interpretation; therefore it is precisely this that we use as the boundary condition for $\mathbf{v}$. The coefficient α in the general case is determined by the magnitude of interaction of particles of a MPF both with a solid boundary and with each other. The value $\alpha = 0$ is due mainly to the interaction of the wall layer of the MPF with the boundary, when it is so large that the particles are not pulled toward the rigid surface. A characteristic feature of the equality $\alpha = 1$ is evidently the complete absence of rotational friction between the particles of the liquid, since for $\mathbf{v}_{|\Gamma} = (1/2)\,(\mathrm{curl}\,\mathbf{v})_{|\Gamma}$ the difference of the translational velocities of the particles that are neighboring with respect to the transverse coordinate does not cause the appearance of distributed surface pairs. In the given case the separate particles of the microelement of the volume do not rotate with respect to the latter, the stress tensor is symmetric, and we are concerned with an ordinary Newtonian liquid.

Using the boundary conditions

$$\mathbf{v}(\pm h) = 0, \quad \mathrm{v}(\pm h) = \frac{\alpha}{2}(\mathrm{curl}\,\mathbf{V})_{|y=\pm h},$$

we solve the system of equations (1)-(2), as a result of which we obtain the following expression for v_x:

$$v_x = v_0\left[1 - \frac{y^2}{h^2} + \frac{2\varkappa(1-\alpha)}{2\mu + \varkappa(2-\alpha)}\,\frac{\mathrm{cth}\,k}{k}\left(\frac{\mathrm{ch}\,ky/h}{\mathrm{ch}\,k} - 1\right)\right], \tag{9}$$

where

$$v_0 = -\frac{dP}{dx}\,\frac{h^2}{2\mu + \varkappa}; \qquad k^2 = \frac{2\mu + \varkappa}{\mu + \varkappa}\,\frac{\varkappa}{\gamma}\,h^2.$$

The study of hydrodynamics and heat exchange in a fluid with a substructure within the framework of the theory of a Newtonian liquid can lead to inaccurate results. For example, in the case of the calculation of the velocity field and temperature during flow of blood with given power of hematocrit H in channels of various cross sections we use a parabolic law of velocity distribution over the cross section. However, actually, in thin channels, we observe a deviation from this law. For shear velocities $\tau \leqslant 250$ sec^{-1} in capillaries with radius $r_0 \leqslant 5 \cdot 10^2$ μm the effective viscosity of the blood depends on the radius [4]; therefore a calculation made according to classical formulas of the velocity field and the temperature in thin capillaries gives inaccurate results. At the same time, using, for example, numerical values of the coefficients $\varkappa$, μ, and γ, obtained in [11] for blood with H = 40%, we can show that for its flow in a plane channel with 2h = 40 μm, taking account of the microstructure within the framework of the theory of a MPF leads to a decrease by a factor of 1.3 in the calculated value of the velocity on the axis for $\alpha = 0$. We emphasize that we are considering the case of a small shear velocity, when the effect of the axial and wall effects is comparatively small [4].

Thermal boundary conditions of the third kind have the form

$$-\lambda\left(\frac{\partial T}{\partial y}\right)_{y=h} = K'\,[T(x,\ h) - T_1]. \tag{10}$$

Substituting (9) into (3) and converting the equation obtained and also (10) into dimensionless form, we obtain

$$\frac{\partial^2\Theta}{\partial\tilde{y}^2} = f(\tilde{y})\,\frac{\partial\Theta}{\partial\tilde{x}}, \tag{11}$$

$$\left(\frac{\partial\Theta}{\partial\tilde{y}}\right)_{\tilde{y}=1} = -\frac{\mathrm{Bi}}{2}\Theta(\tilde{x},\ 1),\quad \left(\frac{\partial\Theta}{\partial\tilde{y}}\right)_{\tilde{y}=0} = 0,\quad \Theta(0,\ \tilde{y}) = 1, \tag{12}$$

where

$$f(\tilde{y}) = \frac{3}{8}\left[1 - \tilde{y}^2 + \frac{2\varkappa(1-\alpha)}{2\mu + \varkappa(2-\alpha)}\,\frac{\operatorname{cth} k}{k}\left(\frac{\operatorname{ch} k\tilde{y}}{\operatorname{ch} k} - 1\right)\right];\quad \tilde{y} = \frac{y}{h};$$

$$\Theta = \frac{T - T_1}{T_0 - T_1};\quad \tilde{x} = \frac{x}{\mathrm{Pe}\,2h};\quad \mathrm{Pe} = \frac{2v_{\mathrm{av}}^{(\mathrm{N})}h}{a};\quad \mathrm{Bi} = \frac{2K'h}{\lambda};$$

$v_{\mathrm{av}}^{(n)}$ is the average velocity of a Newtonian liquid with shear viscosity $\mu + \varkappa/2$ in channel of dimension 2h.

We determine the general Nusselt number [1] $\mathrm{Nu} = 2Kh/\lambda$, where K is the heat-transfer coefficient from the liquid in the channel to the surrounding medium. We can show that

$$\mathrm{Nu} = -\frac{1}{2\bar{\Theta}}\,\frac{d\bar{\Theta}}{d\tilde{x}},\ \text{where}\ \bar{\Theta} = \left[\frac{2}{3} + \frac{2\varkappa(1-\alpha)}{2\mu + \varkappa(2-\alpha)}\cdot\frac{\operatorname{cth} k}{k}\left(\frac{\operatorname{th} k}{k} - 1\right)\right]^{-1}\int_0^1 \Theta f(\tilde{y})\,d\tilde{y}.$$

The problem (11)-(12) was numerically solved on a BÉSM-6 computer by the finite-difference method. In [15] the presence of a dimensional effect in the heat exchange in a MPF is indicated: There is a dependence of Nu on the transverse dimension of the channel. It turned out that for all values of Bi the dimensional effect, whose magnitude is determined by the microstructure and transverse dimension of the channel, proves to have only a small effect on Nu: In the considered range of parameters ($k = 0.1$-5, $\varepsilon = \varkappa/(\mu + \varkappa/2) = 0.4$-$10/7$) with a decrease in h it decreases, but not by more than 2.6%. A small difference of Nu in a MPF and a Newtonian liquid is observed also for the case of thermal boundary conditions of the first and second kinds [16]. The solution of the problem of the exchange between a micropolar fluid and the walls of the channel by thermal dissipation shows that the decrease in Nu calculated within the framework of the MPF theory, in comparison with its value for a Newtonian liquid, does not exceed 18%. All these results indicate that there is considerably less variation of the calculated values of Nu with account of the microstructure of the fluid than in [5-7].

The boundary conditions of the third kind for $\mathrm{Bi} \to \infty$ are converted into boundary conditions of the first kind, for which results of [5-7] are obtained. In this case the heat-transfer coefficient K is the ordinary heat-transfer coefficient appearing in the Newton—Richman law. The difference between the values of Nu in [5-7] and in our results is caused by a different determination of the heat-transfer coefficient, which represents the ratio of

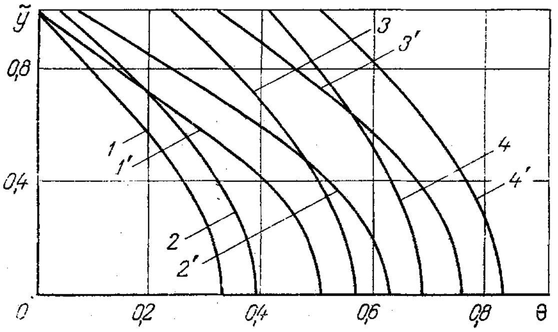

Fig. 1

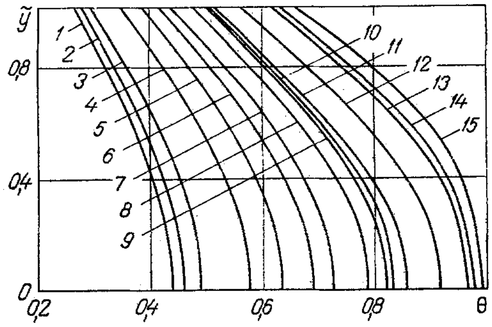

Fig. 2

Fig. 1. Temperature field as a function of Bi for $\tilde{x} = 0.1$: 1, 2, 3, 4) k = 0.1; ε = 10/7; 1', 2', 3', 4') ε = 0; 1, 1') Bi = 10^4; 2, 2') Bi = 20; 3, 3') Bi = 4; 4, 4') Bi = 2.

Fig. 2. Temperature field along the length of the channel as a function of the quantities k and ε for Bi = 2; 1) $\bar{x} = 0.2$; ε = 10/7; k = 0.1; 2) 0.2; 10/7; 1; 3) 0.2; 10/7; 2; 4) 0.2; 0.4; 0.1; 5) 0.2; 0; k – any value; 6) 0.1; 10/7; 0.1; 7) 0.1; 10/7; 2; 8) 0.1; 0.4; 0.1; 9) 0.1; 0; 10) 0.06; 10/7; 0.1; 11) 0.06; 10/7; 2; 12) 0.06; 0; 13) 0.02; 10/7; 0.1; 14) 0.02; 10/7; 2; 15) 0.02; 0.

the thermal flux at the wall to the temperature head. The latter can be determined as the difference of two constant temperatures, for example, that of the wall and that of the input section, and also as the difference in temperature of the wall and the average bulk temperature.

Using the first method of determining the heat-transfer coefficient, the authors of [6-7] showed that calculation of Nu within the framework of the MPF theory for definite values of microstructural parameters leads to its value being one half that of a Newtonian liquid. On this basis, for example, in [6], a conclusion is made concerning the possibility of a considerable decrease in the practical objectives of the rate of heat transfer during flow of a liquid in a channel "by means of attaining still greater micropolarity of a Newtonian solvent." But the analogous result can be more simply attained for the ordinary decrease in flow rate of the same fluid, which is equivalent to an "increase in micropolarity" when for the same pressure gradient there is a decrease in flow rate. In practical objectives it is important to explain the possibility of varying the rate of heat transfer, eliminating from it the contribution due to a decrease in flow rate of the fluid. This enables us to perform the second method of determining the heat-transfer coefficient, which we also used.

In order to determine K with the help of the mean bulk temperature we took into account that the change in magnitude of the thermal flux at the wall can be caused by a change in flow rate of the liquid, and not by the same process of heat exchange. Therefore for the second method of determining the heat-transfer coefficient its magnitude is characterized, first of all, by the relation between the processes of heat transfer by convection and thermal conduction. Thus, the small difference in the values of Nu obtained by us within the framework of the theories of MPF and of a Newtonian liquid indicate that this relation in both cases varies little.

However, the values of the thermal flux at the wall for the same pressure gradients in these cases can strongly differ, which is due to the dependence of the volume flow rate of the liquid on its microstructure. Thus, for example, taking into account the microstructure for k = 0.1, ε = 10/7, α = 0, Bi = 2, and $\tilde{x} = 0.3$ leads to a decrease in the thermal flux at the wall q_w by a factor of 2.3.

The temperature is calculated in such a way that its values in the framework of the theories of MPF and a Newtonian liquid are compared for various volume flow rates, but for the same pressure gradients. In this case we determine the number Pe entering in the dimensionless length $\tilde{x}$, not in terms of the true velocity which the MPF has at the moment being considered, but in terms of the velocity with which a Newtonian liquid with viscosity $\mu + \varkappa/2$ flows in the channel being considered. Otherwise, we would be obliged to make the corresponding calculation in order to explain how the dimensional length of the channel corresponds to some temperature profile.

From Fig. 1 we see that in the entire range of variation of Bi, the temperature fields calculated for the same liquid in the framework of the theory of a Newtonian liquid ($\Theta^{(N)}$) and the theory of a MPF ($\Theta^{(MPF)}$) considerably differ from each other. An analysis of (9) shows that the increase in α for all constants of the remaining parameters is formally equivalent to a decrease in ε for the constants k and α; therefore the problem was solved only for $\alpha = 0$. However, the existence of results of a numerical calculation obtained for different ε enables us when necessary to construct the temperature fields for other values of α. The ratio $\Theta^{(N)}/\Theta^{(MPF)}$ is especially large for large Bi, i.e., for the case in which the third-order boundary conditions are close to the first-order boundary conditions. For example, for $Bi = 10^4$, $\varepsilon = 10/7$, and $k = 0.1$ $\Theta^{(N)}(0.2;\ 0)/\Theta^{(MPF)}(0.2;\ 0) \simeq 2.6$. For the case mentioned above of the flow of blood for a boundary condition of type (5), written for a Cartesian coordinate system, for $Bi = 2$ $\Theta^{(N)}(0.2;\ 0)/\Theta^{(MPF)}(0.2;\ 0) \simeq 1.13$. The temperature at the given point of the channel increases with an increase in k and a decrease in ε (Fig. 2). Thus, the thinner the capillary along which the investigated microstructural fluid flows, the greater will be the difference in the temperature fields calculated by neglecting the microstructure and using the MPF theory.

The considerable difference in temperatures in the initial section of the channel obtained within the framework of the various continuum approaches with consideration of hydrodynamics and heat exchange in a liquid flowing in thin channels makes it necessary in a number of cases to take into account microstructures for calculation of the temperature fields in microstructural fluids.

NOTATION

T is the temperature; $\varkappa$, μ, and γ, the coefficients of viscosity of a micropolar fluid; v_x and ν_z are, respectively, the nonzero components of the vectors of velocity and microrotation; m_{ik}, tensor of the distributed surface pairs (micromoments); α_{ik}, coefficients of the rotational surface friction; n_k, the normal to the surface; λ and a are, respectively, the coefficients of thermal conductivity and of diffusivity of the micropolar fluid.

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